Level:- 5.0 (Second Year) Sem:III

Progressive Education Society's
Modern College of Arts, Science and Commerce (Autonomous),
Shivajinagar, Pune - 5
Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code : 24ScMatU3101
Course Name : Multivariate Calculus

Teaching Scheme: TH: 4 Hours/Week **Credit : 04**
Examination Scheme: CIA : 40 Marks **End-Sem : 60 Marks**

Pre-requisite Courses:

- Basic knowledge of Limits, Continuity, Derivatives and Integration of functions of one variable.
- Basic knowledge of curves and surfaces.

Course Objectives:

- The aim of this course is to study the concept of Limits, Continuity, Partial derivatives of functions of two or three variables.
- To study the methods of evaluation of double and triple integration.
- To study the double integrals in polar form.
- To study the applications of double and triple integration.

Course Outcomes:

On completion of the course, student will be able to understand –

- The evaluation of limits of functions and continuity of functions of two and three variables.
- The use of partial derivatives and differentiability in other subjects.
- The applications of extreme values.
- The methods of evaluating double and triple integration using rectangular coordinates, polar coordinates etc.
- The method to calculate area of bounded region in two dimensional space, surface and volume of closed and bounded regions in three dimensional space.

Course Contents

Unit 1	Limit and Continuity	10 lectures
	• Functions of several variables • Graphs and level curves of functions. • Limits and continuity.	
Unit 2	Partial Derivatives	5 lectures
	• Partial derivatives • Second order partial derivatives.	

	• Mixed derivative theorem. • Partial derivatives of higher order.	
Unit 3	Differentiability	15 lectures
	• Differentiability • Conditions for differentiability. • Increment theorem • Linearization and differentials. • Chain rule.	
Unit 4	Taylor's theorem and extreme values	10 lectures
	• Taylor's theorem for functions of two variables. • Extreme values • First derivative test. • Second derivative test • Lagrange's multipliers method	
Unit 5	Multiple Integrals	20 lectures
	• Double integral over rectangle • Fubini's theorem • Double integrals over general regions. • Change of order of integration. • Double integral in polar form. • Substitution method in double integrals. • Triple integration in rectangular coordinates. • Spherical and cylindrical coordinates in triple integration. • Applications to find area and volumes.	

Reference Books :

1. Mathematical Analysis, (Fifth edition) by S.C. Malik, Savita Arora, New Age International Publishers (2017).
2. A course of Mathematical Analysis, (Revised edition) by Shanti Narayan and P.K. Mittal, S.Chand and Company Pvt. Ltd. (1998)
3. T.M. Apostol, Calculus Volume II (2nd Edition) John Wiley, New York (1967).

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code :24ScMatU3102
Course Name : Lab Course on 24ScMatU3101

Teaching Scheme: PR: 4 Hours/Week **Credit : 02**
Examination Scheme: CIA : 20 Marks **End-Sem : 30 Marks**

Pre-requisite Courses:

- Basic knowledge of Limits, Continuity, Derivatives and Integration of functions of one variable.
- Basic knowledge of curves and surfaces.

Course Objectives:

- The aim of this course is to study the concept of Limits, Continuity, Partial derivatives of functions of two or three variables.
- To study the methods of evaluation of double and triple integration.
- To study the double integrals in polar form.
- To study the applications of double and triple integration.

Course Outcomes:

On completion of the course, student will be able to understand –

- The evaluation of limits of functions and continuity of functions of two and three variables.
- The use of partial derivatives and differentiability in other subjects.
- The applications of extreme values.
- The methods of evaluating double and triple integration using rectangular coordinates, polar coordinates etc.
- The method to calculate area of bounded region in two dimensional space, surface and volume of closed and bounded regions in three dimensional space.

Course Contents

Practical 1 : Limit

Practical 2 : Continuity

Practical 3 : Mixed partial derivative

Practical 4 : Partial derivative

Practical 5 : Differentiation

Practical 6 : Applications of differentials

Practical 7 : Euler's theorem

Practical 8 : Extreme values

Practical 9 : Applications of extreme values

Practical 10 : Lagrange's multipliers method

Practical 11 : Double integration

Practical 12 : Double integration using change of variables

Practical 13 : Change of order of integration

Practical 14 : Triple integration

Practical 15 : Application of integration

OUTLINE OF THE PRACTICALS

Practical 1 : Limits.

1. Problem on direct evaluation of limit
2. Problem on path y = mx
3. Problem on path $y = mx^n$
4. Problem on path $x = my^n$
5. Problem on path other than those in above problems.
6. Problem on evaluation of limits using polar coordinates

Practical 2 : Continuity.

1. Problem on showing function is continuous at a given point (1st problem)
2. Problem on showing function is continuous at a given point (2nd problem)
3. Problem on showing function is not continuous at a given point (1st problem)
4. Problem on showing function is not continuous at a given point (2nd problem)

Practical 3 : Mixed partial derivatives.

1. Problem on finding first order derivative using definition
2. Problem on finding second order derivative at a point using definition
3. Problem on finding mixed partial derivatives at a point using definition
4. Problem on comparing mixed partial derivatives at a point using definition.

Practical 4 : Partial derivatives.

1. Problem based on direct computation of first order partial derivatives (1st problem)
2. Problem based on direct computation of first order partial derivatives (2nd problem)
3. Problem based on direct computation of second order partial derivatives (1st

problem)

4. Problem based on direct computation of second order partial derivatives (2nd problem)
5. Problem based on direct computation of higher order partial derivatives

Practical 5 : Differentiation.

1. Checking whether a function is differentiable using definition. (1st Problem)
2. Checking whether a function is differentiable using definition. (2nd Problem)
3. Checking whether a function is differentiable using definition. (3rd Problem)

Practical 6 : Applications of differentials.

1. Problem on finding approximate value using differentials (1st problem)
2. Problem on finding approximate value using differentials (2nd problem)
3. Problem on finding approximate value using differentials (3rd problem)
4. Problem on finding approximate value using differentials (4th problem)

Practical 7 : Euler's theorems

1. Problem based on identifying homogeneous functions and finding its degree
2. Problem based on Euler's first order theorem (1st Problem)
3. Problem based on Euler's first order theorem (2nd Problem)
4. Problem based on Euler's second order theorem (1st Problem)
5. Problem based on Euler's second order theorem (2nd Problem)

Practical 8 : Extreme values.

1. Finding critical points of given function
2. Finding critical point and a point which has maxima
3. Finding critical point and a point which has minima
4. Finding critical point and a point which has saddle point

Practical 9 : Applications of extreme values.

1. Finding the maximum surface area problem using a function of two variables.
2. Finding minimum expenditure problems using a function of two variables.
3. Maximisation of area problems using a function of two variables.
4. Any other relevant problem related to maximisation and minimisation problem using a function of two variables.

Practical 10 : Lagrange's multipliers method.

1. Maximization problem with one constraint (1st problem)

2. Maximization problem with one constraint (2nd problem)
3. Minimization problem with one constraint (1st problem)
4. Minimization problem with one constraint (2nd problem)

Practical 11 : Double integration

1. Simple problem with both limits as constants
2. Problem with one limit variable and other limit constant and dx dy type problem
3. Problem with one limit variable and other limit constant and dy dx type problem
4. Comparison of double integrals with order of integration changed with both limits constants.

Practical 12 : Double integration using change of variables.

1. Finding Jacobian of transformation for given functions in two variables
2. Problem on change of variables using polar coordinates
3. Problem on change of variables using simple linear transformations
4. Problem on change of variables using arbitrary function.

Practical 13 : Change of order of integration.

1. Change of order of integration involving region bounded by lines
2. Change of order of integration involving region bounded by two parabolas
3. Change of order of integration involving region bounded lines and circles
4. Change of order of integration involving region bounded by arbitrary curves not covered above.

Practical 14 : Triple integration.

1. Problem on triple integration with constant limits
2. Problem on triple integration with one constant and two variable limits
3. Problem on triple integration with related to computation of Jacobian in spherical coordinates
4. Problem on triple integration with related to computation of Jacobian in cylindrical coordinates

Practical 15 : Applications of integration.

1. Finding area in two dimensions
2. Finding surface area in three dimensions
3. Finding volume in three dimensions
4. Any other relevant application of the surface of revolution.

Reference Books :

1. Mathematical Analysis, (Fifth edition) by S.C. Malik, Savita Arora, New Age International Publishers (2017).
2. A course of Mathematical Analysis, (Revised edition) by Shanti Narayan and P.K. Mittal, S.Chand and Company Pvt. Ltd. (1998)
3. T.M. Apostol, Calculus Volume II (2^{nd} Edition) John Wiley, New York (1967).

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU3501
Course Name: Lab Course on Discrete mathematics
Teaching Scheme: PR: 4 Hours/Week **Credit: 2**
Examination Scheme: CIA: 20 Marks **End-Sem : 30 Marks**

Prerequisite Courses: Sets.

Course Objective: The aim of this course is
- To check the equivalence of statements.
- To check the statements for tautology, contradiction and contingency.
- To study methods of proofs.
- To study more generalized counting principles.
- To study linear homogeneous recurrence relation.

Course Outcomes: On completion of the course, students will be able to:
- Use the concept of logic in other mathematical courses.
- Apply different methods of proof.
- Solve problems based on counting principles.
- Solve linear homogeneous recurrence relation.

Course Contents:

Practical 1 : Mathematical induction and Pigeonhole principle
Practical 2 : Inclusion-Exclusion principle
Practical 3 : Arrangement and selection without repetition
Practical 4 : Arrangement and selection with repetition
Practical 5 : Distributions
Practical 6 : Recurrence relations
Practical 7 : Generating functions
Practical 8 : Posets
Practical 9 : Lattices
Practical 10: Graphs
Practical 11: Trees
Practical 12: Isomorphism
Practical 13: Colouring of graphs
Practical 14: Postman/Salesman problems
Practical 15: Shortest path algorithms

Reference Books:
1. Discrete Mathematics and Its Applications by Kenneth H. Rosen, Seventh Edition, McGraw Hill, 2011.

2. Discrete Mathematics Structure by Bernard Kolman, Robert Busby, Sharon Cutler Ross, Nadeem-ur-Rehman, Pearson Education (6th Edition), 2009.
3. Applied Combinatorics by Alan Tucker, Wiley Publication, Fourth Edition 2001

OUTLINE OF THE ABOVE MENTIONED PRACTICALS

Practical 1: Mathematical induction and Pigeonhole principle

1. Problem base on mathematical induction
2. Problem base on strong induction
3. Problem base on pigeonhole principle
4. Problem base on generalized pigeonhole principle
5. Problem base on elegant applications of pigeonhole principle

Practical 2: Inclusion-Exclusion principle

1. Problem base on word problem
2. Problem base on divisibility of integer
3. Problem base on two sets
4. Problem base on three sets
5. Problem base on applications

Practical 3: Arrangement and selection without repetition

1. Problem base on arrangement without repetition
2. Problem base on arrangement without selection
3. Problem base on word problem
4. Problem base on playing cards without replacement
5. Problem base on forming a specific committee without replacement

Practical 4: Arrangement and selection with repetition

1. Problem base on arrangement with repetition
2. Problem base on arrangement with selection
3. Problem base on word problem
4. Problem base on playing cards with replacement
5. Problem base on forming a specific committee without replacement

Practical 5: Distributions

1. Problem base on distinct objects into distinct boxes
2. Problem base on identical objects into distinct boxes
3. Problem base on distinct objects of distinct types into distinct boxes
4. Problem base on distinct objects into boxes where some boxes can be repeated
5. Problem base on distinct objects of distinct types into distinct boxes where some boxes can be repeated

Practical 6 : Recurrence relations

1. Problem base on sequence
2. Problem base on compound interest
3. Problem base on rabbits and Fibonacci numbers
4. Problem base on tower of Hanoi
5. Problem base on bit strings

Practical 7 : Generating functions

1. Problem base on power series
2. Problem base on binomial theorem

3. Problem base on exponential generating functions
4. Problem base on using generating functions to solve recurrence relations
5. Problem base on proving identities

Practical 8 : Posets

1. Problem base on counting of reflexive relations
2. Problem base on counting of antisymmetric relations
3. Problem base on counting of transitive relations
4. Problem base on diagrams of finite posets up to similarity
5. Problem base on counting of finite posets

Practical 9 : Lattices

1. Problem base on diagrams of finite lattice up to similarity
2. Problem base on counting of lattices up to four elements
3. Problem base on counting of lattices up to ten elements containing two reducible elements
4. Problem base on counting of lattices up to ten elements containing three reducible elements
5. Problem base on application

Practical 10: Graphs

1. Problem base on degree and vertices
2. Problem base on handshaking theorem
3. Problem base on complete graph
4. Problem base on bipartite graph
5. Problem base on applications

Practical 11: Trees

1. Problem base on identification of tree
2. Problem base on binary tree
3. Problem base on height of tree
4. Problem base on balanced tree
5. Problem base on application

Practical 12: Isomorphism

1. Problem base on posets
2. Problem base on lattices
3. Problem base on graphs
4. Problem base on trees
5. Problem base on application

Practical 13: Colouring of graphs

1. Problem base on determining the chromatic number of a graph
2. Problem base on four colour theorem
3. Problem base on chromatic numbers for complete graph
4. Problem base on chromatic numbers for complete bipartite graph
5. Problem base on cycle with n vertices
6. Problem base on applications of graph colourings

Practical 14: Postman/Salesman problems

1. Problem base on Chinese postman problem in an undirected graph
2. Problem base on Chinese postman problem in a directed graph
3. Problem base on travelling salesman problems in an undirected graph
4. Problem base on travelling salesman problems in a directed graph
5. Problem base on application

Practical 15: Shortest path algorithms

1. Problem base on simple network using Dijkstra's algorithm
2. Problem base on moderate network using Dijkstra's algorithm
3. Problem base on difficult network using Dijkstra's algorithm
4. Problem base on Warshall algorithm
5. Problem base on applications.

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Second Year of B.Sc.
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Course Code: 24ScMatU3901
Course Name: Ancient Indian Mathematics
Teaching Scheme: TH: 2 Hours/Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Pre-requisites: Mathematics up to SSC level

Course Objectives: To study

- Numbers, fractions and geometry in the Vedas
- Decimal nomenclature of numbers in the Vedas
- Zero and Infinity
- Simple constructions from Sulba-sutras
- The development of the decimal place value system which resulted in a simplification of all arithmetical operations
- Linguistic representation of numbers
- Important texts of Indian mathematics
- Proofs in Indian Mathematics
- Brief introduction to the development of algebra, trigonometry and calculus
- Overview of development of Mathematics in India during the ancient and early classical Period (till 500 CE), later classical period (500-1250), medieval period (1250-1750) and modern periods (1750- present

Course Outcomes:

On completion of the course, student will be able to:-

- Understand glorious history of Mathematics and valuable contribution by Indian mathematicians to the world
- To Research on the Indian Knowledge System.

Course Contents:

Unit 1	Mathematics in the Vedas and Śulva Sūtras	2 Hours
Unit 2	Pāṇini's *Aṣṭādhyāyī*	1 Hours
Unit 3	Piṅgala's *Chandaḥśāstra*	1 Hours
Unit 4	Mathematics in the Jaina Texts	1 Hours

Unit 5	Development of Place Value System	1 Hours
Unit 6	*Āryabhaṭīya* of Āryabhaṭa	2 Hours
Unit 7	*Brāhmasphuṭasiddhānta* of Brahmagupta	2 Hours
Unit 8	Bakṣālī Manuscript	1 Hours
Unit 9	*Gaṇitasārasaṅgraha* of Mahāvīra	1 Hours
Unit 10	Development of Combinatorics	2 Hours
Unit 11	*Līlāvatī* of Bhāskarācārya	2 Hours
Unit 12	*Bījagaṇita* of Bhāskarācārya	2 Hours
Unit 13	*Gaṇitakaumudī* of Nārāyaṇa Paṇḍita	2 Hours
Unit 14	Magic Squares	1 Hours
Unit 15	Kerala School of Astronomy and Development of Calculus	3 Hours
Unit 16	Trigonometry and Spherical Trigonometry	2 Hours
Unit 17	Proofs in Indian Mathematics	2 Hours
Unit 18	Mathematics in Modern India	2 Hours
	Total No. of Hours	**30**

Reference Books:

1. B. Datta and A. N. Singh, History of Hindu Mathematics, 2 Parts, Lahore, 1935, 1938; Reprint, Asia Publishing House, Bombay 1962; Reprint, Bharatiya Kala Prakashan, Delhi 2004.
2. C. N. Srinivasa Nagar, History of Indian Mathematics, The World Press, Calcutta, 1967.
3. T. A. Saraswati Amma, Geometry in Ancient and Medieval India, Motilal Banarsidass, Varanasi, 1979.
4. S. Balachandra Rao, Indian Mathematics and Astronomy: Some Landmarks, 3rd Ed. Bhavan's Gandhi Centre, Bangalore, 2004.
5. G. G. Emch, M. D. Srinivas and R. Sridharan, Eds., Contributions to the History of Mathematics in India, Hindustan Book Agency, Delhi, 2005.
6. C. S. Seshadri, Ed., Studies in History of Indian Mathematics, Hindustan Book Agency, Delhi, 2010.
7. P. P. Divakaran, The Mathematics of India Concepts Methods Connections, Hindustan Book Agency 2018. Rep Springer New York, 2018.
8. Gaṇitayuktibhāṣā (c.1530) of Jyeṣṭhadeva (in Malayalam), Ed. with Tr. by K. V. Sarma with Explanatory Notes by K. Ramasubramanian, M. D. Srinivas and M. S. Sriram, 2 Volumes, Hindustan Book Agency, Delhi, 2008.

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU3301
Course Name: Numerical Methods and its applications
Teaching Scheme: TH: 2 Hours/Week **Credits: 2**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Prerequisites: Equations, Functions, Polynomials and Differential Equations.

Course Objectives: To Study

- Methods for solution of algebraic and transcendental equations, and a system of linear equations.
- Curve fitting using a line, a parabola, a power function and an exponential function.
- Polynomial approximation and interpolation.
- Numerical differentiation and integration.
- Numerical methods to solve Ordinary Differential Equations of first degree and first order.

Course Outcomes: On completion of the course, student will be able to:-

- Find the approximate real root of the equation and solve a system of linear equations numerically.
- Approximate a known or unknown function by a polynomial with desired accuracy.
- Find numerical differentiation and evaluate definite integrals numerically.
- Solve first order and first degree Ordinary Differential Equations.

Course Contents:

Unit 1	Solutions of Equations	06 lectures
	• Bisection method. • Regula falsi method. • Newton-Raphson method.	
Unit 2	Interpolation	10 lectures
	• Operators Δ, ∇, E and their relations • Differences: forward, backward, divided • Fundamental theorem of difference calculus • Newton-Gregory formula for forward/backward interpolation. • Lagrange's interpolation formula.	
Unit 3	Integration	08 lectures
	• General quadrature formula. • Trapezoidal rule. • Simpson's $1/3^{rd}$ rule. • Simpson's $3/8^{th}$ rule.	
Unit 4	Applications	06 lectures
	• Euler's method. • Euler's modified methods • Largest eigenvalue using power method	

Reference Books:

1. Introductory Methods of Numerical Analysis by S.S. Sastry, Prentice Hall of India Fifth Edition, 2012.
2. Finite differences and Numerical Analysis by H.C. Saxena, S. Chand, 2010.
3. A textbook of Computer Based Numerical and Statistical Techniques by A. K. Jaiswal and Anju Khandelwal, New Age International Publishers, 2009.
4. Linear Algebra and Its Applications by David C. Lay, Steven R. Lay, Judi J. McDonald, Sixth Edition, 2022.

Progressive Education Society's
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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU3302
Course Name: Lab Course on 24ScMat3301
Teaching Scheme: PR: 4 Hours/Week **Credit: 2**
Examination Scheme: CIA: 20 Marks **End-Sem : 30 Marks**

- **Prerequisite Courses:** Equations, Functions, Polynomials and Differential Equations.

Course Objective: To study
- Solution of equations
- Solution of the system of linear equations.
- Curve fitting.
- Polynomial approximation and interpolation.
- Numerical differentiation and integration.
- Solving Ordinary Differential Equations

Course Outcomes: On completion of the course, students will be able to: -
- Find approximate real root of an equation
- Solve a system of linear equations numerically.
- Approximate a function by a polynomial with desired accuracy.
- Interpolate an equally spaced and unequally spaced data.
- Find numerical differentiation and integration.
- Solve Ordinary Differential Equations numerically.

Course Contents:

Practical 1: Bisection method
Practical 2: Regula Falsi method and Secant method
Practical 3: Newton Raphson Method
Practical 4: Aitken's Δ^2-process
Practical 5: Operators and their relations
Practical 6: Newton's forward interpolation formula
Practical 7: Newton's Backward interpolation formula
Practical 8: Lagrange's interpolation formula
Practical 9: Newton's Divided difference formula
Practical 10: Numerical Differentiation
Practical 11: Numerical Integration 1
Practical 12: Numerical Integration 2
Practical 13: Runge Kutta method
Practical 14: Gauss elimination method
Practical 15: Least square method / Curve fitting

Reference Books:

1) Introductory Methods of Numerical Analysis by S.S. Sastry, Prentice Hall of India Fifth Edition, 2012.
2) Finite differences and Numerical Analysis by H.C. Saxena, S. Chand, 2010.
3) A textbook of Computer Based Numerical and Statistical Techniques by A. K.Jaiswal and Anju Khandelwal, New Age International Publishers, 2009.
4) Computer oriented Numerical Methods by V. Rajaraman, PHI Learning Private Limited, New Delhi, Third Edition, 2011.
5) Numerical Methods by E Balagurusamy, Tata McGraw Hill Education Private Limited, New Delhi, Reprint 2012.

OUTLINE OF THE ABOVE MENTIONED PRACTICALS

Practical 1: Bisection Method

1. Example on nonlinear equation with given interval
2. Example on nonlinear equation without given interval
3. Transcendental equation with initial root
4. Transcendental equation without initial root
5. Practice example

Practical 2: Regula Falsi Method and Secant Method

1. Nonlinear with interval by Regula Falsi method
2. Nonlinear without interval by Regula Falsi method
3. Nonlinear with interval by Secant method
4. Nonlinear without interval by Secant method
5. Example on Regula Falsi method

Practical 3: Newton-Raphson Method

1. Example on nonlinear equation with initial root
2. Example on nonlinear equation without initial root
3. Transcendental equation with initial root
4. Transcendental equation without initial root
5. Transcendental equation example

Practical 4: Iteration Method

1. Example on iteration method with initial root
2. Example on iteration method without initial root
3. Example using Aitken's Δ^2-process
4. Example using Aitken's Δ^2-process
5. Example using Aitken's Δ^2-process

Practical 5: Operators and Their Relations

1. Relation between ∇ and E
2. Relation between Δ and ∇
3. Examples on average operatior
4. Examples on Δ, ∇, E
5. Examples on Δ, ∇, E

Practical 6: Newton's Forward Interpolation Formula

1. Example to construct forward table
2. Example base on given data
3. Example base on given $x_{0,}$ h and y(x)
4. Practice example
5. Practice example

Practical 7: Newton's Backward Interpolation Formula

1. Example to construct backward table
2. Example base on given data
3. Example base on given $x_{0,}$ h and y(x)
4. Practice example
5. Practice example

Practical 8: Lagrange's Interpolation Formula

1. Example to find y at x using given data equally spaced
2. Example to find the formula for y(x)
3. Example to find y at x using given data unequally spaced
4. Example to find the formula for y(x)
5. Practice example

Practical 9: Newton's Divided Difference Formula

1. Example to find y at x using given data equally spaced
2. Example to find the formula for y(x)
3. Example to find y at x using given data unequally spaced
4. Example to find the formula for y(x)
5. Practice example

Practical 10: Numerical Differentiation

1. Example to find $\frac{dy}{dx}$ at $x = x_0$
2. Example to find $\frac{dy}{dx}$ and $\frac{d2}{dx}$ at $y = x_0$
3. Example to find $\frac{dy}{dx}$ and $\frac{d2}{dx}$ at $y = x_1$
4. Practice example on velocity and acceleration
5. Practice example

Practical 11: Numerical Integration - I

1. Example on trapezoidal rule with given data
2. Example on trapezoidal rule without given data
3. Example on Simpson's $1/3^{rd}$ rule with given data
4. Example on Simpson's $1/3^{rd}$ rule without given data
5. Practice example

Practical 12: Numerical Integration - II

1. Example on Simpson's $3/8^{th}$ rule with given data
2. Example on Simpson's $3/8^{th}$ rule without given data
3. Example base on velocity and displacement
4. Example on trapezoidal rule
5. Example on Simpson's $1/3^{rd}$ rule

Practical 13: Runge-Kutta Method

1. Example on 2^{nd} order
2. Example on 2^{nd} order

3. Example on 4^{th} order
4. Example on 4^{th} order
5. Practice example

Practical 14: Gauss seidel Elimination Method

1. Example on Gauss seidel elimination up to 2 iterations
2. Example on Gauss seidel elimination up to 3 iterations
3. Example on Gauss seidel elimination up to 4 iterations
4. Practice example
5. Practice example

Practical 15: Least Square Method / Curve Fitting

1. Example on fitting the data with linear polynomial
2. Example on fitting the data with quadratic function
3. Example on fitting the data with exponential function
4. Example on fitting the data with power function
5. Example on fitting the data with polynomials

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Course Code: 24ScMatU3401
Course Name: Financial Mathematics
Teaching Scheme: TH: 2 Hours/Week **Credit: 2**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Prerequisites: Basic knowledge of derivatives, matrices, linear equations, elementary statistics, principles of economics

Course Objectives: The aim of this course is to study

- Computational skills necessary to implement pricing, hedging, trading and risk management tools.
- Mathematical modelling techniques used in finance.
- How to improve your personal skills, including logical reasoning, quantitative analysis.
- How to apply mathematical concepts in financial models, economics.

Course Outcomes: On completion of the course, student will able to

- Apply the concepts of financial mathematics in Actuarial science to study of assessing risk in insurance and finance also in, Data mining, Data Science, Econometrics.
- Construct a model of the market.
- Determine supply and demand functions.
- Find and classify critical points.
- Solve profit maximization using derivatives.
- Design the quantitative methodologies and techniques useful in investment banks and other financial institutions.

Course Contents:

Unit 1	**Mathematical models in economics**	**06 Lectures**
	• Introduction • A model of the market	

	• Market equilibrium and excise tax.	
Unit 2	**The elements of finance and the cobweb model**	**08 Lectures**
	• Interest and capital growth • Income generation • The interval of compounding	
Unit 3	**Introduction to optimization:**	**08 Lectures**
	• Profit maximization • Critical points • Profit maximization using derivatives	
Unit 4	**Portfolios**	**08 Lectures**
	• Introduction to portfolios • Making money with matrices • A two-industry 'economy', arbitrage portfolios and state prices	
	Total Lectures	**30 Lectures**

Reference Books:

1. Introduction to Financial Mathematics by Arash Fahim, Florida State University, 2019.
2. Mathematical Economics by Edward T. Dowling, Schaum's outline Series. McGraw Hill International Edition, Second Edition.
3. Calculus of Finance by Amber Habib, University Press, 2011.
4. Mathematics for Economics and Finance Methods and Modelling by Martin Anthony and Norman Biggs, Cambridge University Press, 2012

Level:- 5.0 (Second Year) Sem:IV

Progressive Education Society's
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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4101
Course Name: Linear Algebra
Teaching Scheme: TH: 4 Hours/Week **Credits: 4**
Examination Scheme: CIA: 40 Marks **End-Sem : 60 Marks**

Prerequisites: Basic knowledge of function, matrices and system of linear equations.

Course Objectives: To study,
- Basics of group theory.
- The dimension of a vector space.
- Inner product on a vector space.
- Matrix representation of a linear transformation.

Course Outcomes: Student will be able to,
- Understand algebraic structures.
- Check whether a given set is a basis of a vector space.
- Understand the usual vector spaces.
- Understand use of inner product spaces.

Course Contents :

Unit 1	Vector Spaces	20 lectures
	• Simple algebraic structures • Vector space • Subspace. • Linear span, Linear dependence and independence. • Basis and dimension. • Row space and Column space of a matrix • Basis and dimension of solution space of homogeneous systems of linear equations.	
Unit 2	Linear Transformations	15 lectures

	• Linear Transformations • Properties of linear transformation. • Matrix of a linear transformation. • Kernel and range of a linear transformation. • Rank Nullity theorem	
Unit 3	Inner product spaces	15 lectures
	• Inner product, Norm, Angle and Distance between two vectors. • Parallelogram Law and Pythagorean Law in inner product spaces. • Orthogonal projection and Orthogonal basis. • Orthonormal basis • Gram Schmidt process to obtain orthonormal basis.	
Unit 4	Eigenvalues and Eigenvectors	10 lectures
	• Eigenvalues and Eigenvectors of a linear transformation. • Diagonalization of a matrix.	
	Total Lectures	**60 lectures**

Reference Books :

1. Schaum's outline of Theory and Problems of Linear Algebra by Seymour Lipschitz and Marc Lars Lipson, 3rd edition, 1968.
2. Introduction to Linear Algebra by Gilbert Strang, Wellesley-Cambridge Press,U.S.; 6th edition,2023.
3. Matrix and Linear Algebra aided with Matlab by Kanti Bhushan Datta, PHI Learning Pvt. Ltd., New Delhi, Second edition, 2012.
4. Linear Algebra, A geometric approach by S. Kumaresan, PHI Learning Pvt. Ltd., New Delhi, 2009.
5. Linear Algebra and its Applications by David C. Lay, Steven Lay, Judy McDonald Sixth Edition, 2022.
6. Elementary Linear Algebra by Howard Anton and Chris Rorres, Wiley Publications, 2014.

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4102
Course Name: Lab Course on 24ScMatU4101
Teaching Scheme: PR: 4 Hours/Week **Credit: 2**
Examination Scheme: CIA: 20 Marks **End-Sem : 30 Marks**

Prerequisites: Basic knowledge of function, matrices and system of linear equations.

Course Objectives: To study,

- Basics of group theory.
- The dimension of a vector space.
- Inner product on a vector space.
- Matrix representation of a linear transformation.

Course Outcomes: Student will be able to,

- Understand algebraic structures.
- Check whether a given set is a basis of a vector space.
- Understand the usual vector spaces.
- Understand use of inner product spaces.

Course Contents:

Practical 1 : Vector spaces
Practical 2 : Subspaces
Practical 3 : Linear dependence and independence
Practical 4 : Basis
Practical 5 : Row space and column space
Practical 6 : Linear transformations
Practical 7 : Kernel and image
Practical 8 : Eigenvalues and eigenvectors
Practical 9 : Diagonalization
Practical 10 : Cayley Hamilton theorem
Practical 11 : Inner product spaces
Practical 12 : Geometry in Inner Product spaces
Practical 13 : Orthogonal projections
Practical 14 : Gram-Schmidt process
Practical 15 : LU decomposition

OUTLINE OF THE ABOVE MENTIONED PRACTICALS

Practical 1 : Vector spaces

1. Problem based on Vector spaces (example 1)
2. Problem based on Vector spaces (example 2)
3. Problem based on Vector spaces (example 3)
4. Check whether V is a vector space. (example 1)
5. Check whether V is a vector space. (example 2)

Practical 2 : Subspaces

1. Problem based on subspace of a vector space
2. Example of subset which is not a subspace of a vector space
3. Determining all subspaces of R^2.
4. Subspaces of R^3.

Practical 3 : Linear dependence and independence

1. Example of linear independent set in R^2.
2. Example of linear independent set in R^3.
3. Example of linear dependent set in R^2
4. Example of linear dependent set in R^3.

Practical 4 : Basis

1. Example of basis in R^2
2. Example of a subset which is not basis in R^2
3. Example of basis in R^3
4. Example of a subset which is not basis in R^3

Practical 5 : Row space and column space

1. Find row space of 3 x 3 matrix
2. Find row space of 3 x 5 matrix
3. Find row space of 5 x 3 matrix
4. Find row space and column space of 3 x 3 matrix and compare them.
5. Find row space and column space of 3 x 5 matrix and compare them.

Practical 6 : Linear transformations

1. Show $T: R^2 \to R^2$ is a linear transformation.
2. Show $T: R^2 \to R^3$ is a linear transformation.
3. Show $T: R^3 \to R^3$ is a linear transformation.
4. Check whether $T: R^2 \to R^2$ is a linear transformation.
5. Check whether $T: R^2 \to R^3$ is a linear transformation.

Practical 7 : Kernel and image

1. Find the kernel space of a linear transformation $T: R^2 \to R^2$
2. Find the basis of kernel space of a linear transformation $T: R^4 \to R^3$
3. Find the range space of a linear transformation $T: R^2 \to R^3$
4. Find the range space of a linear transformation $T: R^3 \to R^2$
5. Verify Rank Nullity theorem.

Practical 8 : Eigenvalues and eigenvectors

1. Find eigenvalues of 2 x 2 matrix with distinct real eigenvalues
2. Find eigenvalues of 3 x 3 matrix with distinct real eigenvalues
3. Find eigenvalues of 2 x 2 matrix with repeated real eigenvalues
4. Find eigenvalues of 3 x 3 matrix with one repeated real eigenvalue
5. Find eigenvalues of 2 x 2 matrix with complex eigenvalues
6. Find the characteristic polynomial and largest eigenvalue.

Practical 9 : Diagonalization

1. Diagonalisation of a 2 x 2 matrix
2. Diagonalisation of a 3 x 3 matrix

3. Find $P^{-1}AP$ for a given matrix of size 2x2 matrix.
4. Find $P^{-1}AP$ for a given matrix of size 3x3 matrix.
5. Find A^k given a 2 x 2 matrix using diagonalisation process.

Practical 10 : Cayley Hamilton theorem

1. Verification of Cayley Hamilton theorem of 2 x 2 matrix.
2. Verification of Cayley Hamilton theorem of 3 x 3 matrix.
3. Finding inverse of matrix using Cayley Hamilton theorem
4. Find power of a matrix using Cayley Hamilton theorem.

Practical 11 : Inner product spaces

1. Show that given product is an inner product space
2. Show that given product is an inner product space
3. Check whether the given product is an inner product space.
4. Problem based on computing inner product and angle between two vectors in V

Practical 12 : Geometry in Inner product space

1. Problem based on Pythagorean Law or Parallelogram law.
2. Problem based on Parallelogram law.
3. Problem based on geometry using inner product.
4. Problem to show or check orthogonal vectors

Practical 13 : Orthogonal projections.

1. Problem based on finding orthogonal projection of a vector along another vector.
2. Problem based on finding orthogonal projection of a vector along a subspace
3. Problem based on finding basis of orthogonal complement.
4. Problem based on finding orthogonal projections of two vectors along given subspace.

Practical 14 : Gram-Schmidt process

1. Example of two vectors in R^2 (example 1)

2. Example of two vectors in R^2 (example 2)
3. Example of three vectors in R^3 (example 1)
4. Example of two vectors in R^3 (example 2)
5. Example of three polynomials in P_2

Practical 15 : LU Decomposition.

1. Five problems based on LU decomposition.

Reference Books :

1. Schaum's outline of Theory and Problems of Linear Algebra by Seymour Lipschitz and Marc Lars Lipson, 3rd edition, 1968.
2. Introduction to Linear Algebra by Gilbert Strang, Wellesley-Cambridge Press,U.S.; 6th edition,2023.
3. Matrix and Linear Algebra aided with Matlab by Kanti Bhushan Datta, PHI Learning Pvt. Ltd., New Delhi, Second edition, 2012.
4. Linear Algebra, A geometric approach by S. Kumaresan, PHI Learning Pvt. Ltd., New Delhi, 2009.
5. Linear Algebra and its Applications by David C. Lay, Steven Lay, Judy McDonald Sixth Edition, 2022.
6. Elementary Linear Algebra by Howard Anton and Chris Rorres, Wiley Publications, 2014.

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4501
Course Name: Lab Course on Computational Geometry

Teaching Scheme: TH: 4 Hours/Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Prerequisites: Matrix operations, planar graphs.

Course Objectives:

1) Students will learn about basic two-dimensional transformations such as translation, rotation, scaling, and reflection.
2) Students will study transformations in three-dimensional space, including rotation, scaling, translation, and shearing.
3) Students will learn about different types of projections, such as orthographic and perspective projections.
4) Students will study the mathematical foundations and algorithms used to generate plane curves, such as circle, ellipse, parabola, and hyperbola.
5) Students will explore the interconnections between transformations, projections, and curve generation.

Course Outcomes: On completion of the course

1) Students will be able to effectively apply translation, scaling, rotation, and reflection transformations to 2D geometric shapes and objects.
2) Students will be able to visualize the effects of 3D transformations in a computational environment.
3) Students will demonstrate the ability to apply different projection techniques such as orthographic and perspective projections to map 3D objects onto 2D planes.
4) Students will understand the mathematical foundations and applications of planar curves in fields like animation, CAD, and graphic design.
5) Students will develop algorithms to perform geometric operations, including transformations of curves and surfaces, and apply them to real-world applications like object rendering and modeling.
6) Students will demonstrate the ability to apply computational geometry techniques to real-world computer graphics, 3D rendering, virtual environment creation, modelling and simulation.

Course Contents

Practical 1: Two dimensional transformations
Practical 2: Transformation of polygons
Practical 3: Combined transformations
Practical 4: Area of a plane figure and solid body transformations
Practical 5: Homogeneous coordinates and translation
Practical 6: Rotation about arbitrary point
Practical 7: Reflection through arbitrary line
Practical 8: Three dimensional transformations
Practical 9: Reflection through a local axis
Practical 10: Reflection through planes parallel to coordinate planes
Practical 11: Orthographic and axonometric projections
Practical 12: Oblique projections
Practical 13: Perspective projections
Practical 14: Generation of points on a circle and an ellipse
Practical 15:Generation of an arc of parabola and hyperbola

Reference Books:

1) D.F.Rogers, J. A. Adam, Mathematical elements for computer graphics, Tata Mc Graw Hill International Edition, Second edition, 2002.
2) Donald Hearn and M Pauline Baker, Warren Carithers, Computer Graphics with Open GL, Pearson (4th edition), 2014.
3) Schaum's Outline Series, Computer Graphics, tata Mc Graw Hill International Edition, 2015.

Outline of each practical

Practical 1: Two Dimensional Transformations

1) Examples on all types of scaling
2) Examples on all types of shearing
3) Examples on all types of reflection
4) Examples on rotation with different θ
5) Examples on uniform scaling

Practical 2: Transformation of Polygons

1) Example on transformation of line segment
2) Example on transformation of square
3) Examples on slope and y-intercept of transformed line
4) Example on intersection point of transformed intersecting lines
5) Example on midpoint of transformed line

Practical 3: Combined Transformations

Five questions on various combinations of transformations: scaling, uniform scaling, shearing, reflection and rotation to obtain a combined transformation matrix.

Practical 4: Area of a plane figure and solid body transformations

1) Example on area of transformed square
2) Example on area of transformed circle
3) Example on solid body transformation
4) Example on area of transformed figure
5) Example on solid body transformation

Practical 5: Homogeneous coordinates and translation

Five questions on various combinations of transformations: scaling, shearing, reflection, rotation, translation and overall scaling to obtain combined transformation matrix and applying on object.

Practical 6: Rotation about arbitrary point

1) Example on rotation about an arbitrary point
2) Example on rotation about midpoint of segment
3) Example on rotation about centroid og triangle
4) Practice example
5) Practice example

Practical 7: Reflection through arbitrary line

1) Example on reflection through an arbitrary line $ax+by+c=0$
2) Example on reflection through line passing through origin
3) Example on reflection through line parallel to X-axis
4) Example on reflection through line parallel to Y-axis

5) Practice example

Practical 8: Three dimensional transformation

Five questions on various combinations of transformations: scaling, shearing, reflection, rotation, translation and overall scaling to obtain combined transformation matrix and applying on three dimensional objects.

Practical 9: Rotation through a local axis

1) Example on rotation through local *X*-axis through an angle θ
2) Example on rotation through local *Y*-axis through an angle θ
3) Example on rotation through local *Z*-axis through an angle θ
4) Example on rotation through local *X*-axis through an angle -θ
5) Example on rotation through local *Y*-axis through an angle -θ

Practical 10: Reflection through planes parallel to coordinate planes

1) Example on reflection through *XY*-plane
2) Example on reflection through *XZ*-plane
3) Example on reflection through *YZ*-plane
4) Practice example
5) Practice example

Practical 11: Orthographic and axonometric projections

1) Two examples on various combinations of transformations: scaling, shearing, reflection, rotation, translation and orthographic projections in three dimensional transformations to obtain a combined transformation matrix and apply it on objects.
2) Three/four examples on trimetric, dimetric and isometric projections.

Practical 12: Oblique and perspective projections

1) Example on cavalier projection
2) Example on cabinet projection
3) Examples of various combinations of transformations: scaling, shearing, reflection, rotation, translation and perspective projections in three dimensional transformations to obtain a combined transformation matrix and apply it on the object.

Practical 13: Generation of an circle

1) Example on generation of equally spaced points on standard circle.
2) Example on generation of equally spaced points in the first quadrant of standard circle.
3) Example on generation of equally spaced points in first and second quadrant of standard circle.
4) Example on generation of equally spaced points on general circle
5) Example on generation of equally spaced points on general circle

Practical 14: Generation of a ellipse

1) Example on generation of equally spaced points on standard ellipse.
2) Example on generation of equally spaced points in the first quadrant of standard ellipse.
3) Example on generation of equally spaced points in first and second quadrant of standard ellipse.
4) Example on generation of equally spaced points on general ellipse.
5) Example on generation of equally spaced points on general ellipse.

Practical 15:Generation of an arc of parabola and hyperbola

1) Example on generation of equally spaced points on arc of parabola when range for *x*-coordinate is given.
2) Example on generation of equally spaced points on arc of parabola when range for *y*-coordinate is given.
3) Example on generation of equally spaced points on arc of hyperbola when range for *x*-coordinate is given.
4) Example on generation of equally spaced points on arc of hyperbola when range for *y*-coordinate is given.
5) Practice example

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4301
Course Name: Linear Algebra
(Section - I)
Teaching Scheme: TH: 2 Hours/Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Semester: 30 Marks**

Prerequisites: Basic knowledge of function, matrices and system of linear equations.

Course Objectives: To study,
- The dimension of a vector space.
- Inner product on a vector space.
- Matrix representation of a linear transformation.

Course Outcomes: Student will be able to,
- Check whether a given set is a basis of a vector space.
- Understand the usual vector spaces.
- Understand use of inner product spaces.

Course Contents :

Unit 1	Vector Spaces	10 lectures
	• Definition and Examples, Subspaces. • Linear span, Linear dependence and independence. • Basis and dimension. • Basis and dimension of solution space of homogeneous systems of linear equations.	
Unit 2	Linear Transformations	10 lectures
	• Definition and examples, Properties of linear transformation. • Matrix of a linear transformation. • Kernel and range of a linear transformation. • Eigenvalues and Eigenvectors of a linear transformation.	
Unit 3	Inner product spaces	10 lectures

	• Inner product, Norm, Distance between two vectors. • Parallelogram Law and Pythagorean Law in inner product spaces. • Orthogonal and Orthogonal sets.	
Total Lectures		**30 lectures**

Reference Books :

1. Schaum's outline of Theory and Problems of Linear Algebra by Seymour Lipschitz and Marc Lars Lipson, 3rd edition, 1968.
2. Introduction to Linear Algebra by Gilbert Strang, Wellesley-Cambridge Press,U.S.; 6th edition,2023.
3. Matrix and Linear Algebra aided with Matlab by Kanti Bhushan Datta, PHI Learning Pvt. Ltd., New Delhi, Second edition, 2012.
4. Linear Algebra, A geometric approach by S. Kumaresan, PHI Learning Pvt. Ltd., New Delhi, 2009.
5. Linear Algebra and its Applications by David C. Lay, Steven Lay, Judy McDonald Sixth Edition, 2022.
6. Elementary Linear Algebra by Howard Anton and Chris Rorres, Wiley Publications, 2014.

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4302
Course Name: Lab Course on 24ScMatU4301
(Section -II)

Teaching Scheme: 4 Hours/ Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Semester: 30 Marks**

Prerequisites: Basic knowledge of function, matrices and system of linear equations.

Course Objectives: To study,

- The dimension of a vector space.
- Inner product on a vector space.
- Matrix representation of a linear transformation.

Course Outcomes: Student will be able to,

- Check whether a given set is a basis of a vector space.
- Understand the usual vector spaces.
- Understand use of inner product spaces.

Course Contents :

Practical 1 : Vector spaces
Practical 2 : Subspaces
Practical 3 : Linear dependence and independence
Practical 4 : Basis
Practical 5 : Row space and column space
Practical 6 : Linear transformations
Practical 7 : Kernel and image
Practical 8 : Eigenvalues and eigenvectors of 2 x 2 matrix
Practical 9 : Eigenvalues and eigenvectors of 3 x 3 matrix
Practical 10 : Diagonalization
Practical 11 : Cayley Hamilton theorem
Practical 12 : Inner product spaces
Practical 13 : Orthogonal and Orthonormal sets
Practical 14 : Gram-Schmidt process in R^2

Practical 15 : Gram-Schmidt process in R^3

Reference Books :

7. Schaum's outline of Theory and Problems of Linear Algebra by Seymour Lipschitz and Marc Lars Lipson, 3rd edition, 1968.
8. Introduction to Linear Algebra by Gilbert Strang, Wellesley-Cambridge Press,U.S.; 6th edition,2023.
9. Matrix and Linear Algebra aided with Matlab by Kanti Bhushan Datta, PHI Learning Pvt. Ltd., New Delhi, Second edition, 2012.
10. Linear Algebra, A geometric approach by S. Kumaresan, PHI Learning Pvt. Ltd., New Delhi, 2009.
11. Linear Algebra and its Applications by David C. Lay, Steven Lay, Judy McDonald Sixth Edition, 2022.
12. Elementary Linear Algebra by Howard Anton and Chris Rorres, Wiley Publications, 2014.

OUTLINE OF THE ABOVE MENTIONED PRACTICALS

Practical 1 : Vector spaces

1. Problem based on Vector spaces (example 1)
2. Problem based on Vector spaces (example 2)
3. Problem based on Vector spaces (example 3)
4. Check whether V is a vector space. (example 1)
5. Check whether V is a vector space. (example 2)

Practical 2 : Subspaces

1. Problem based on subspace of a vector space
2. Example of subset which is not a subspace of a vector space
3. Determining all subspaces of R^2.
4. Subspaces of R^3.

Practical 3 : Linear dependence and independence

1. Example of linear independent set in R^2.
2. Example of linear independent set in R^3.
3. Example of linear dependent set in R^2
4. Example of linear dependent set in R^3.

Practical 4 : Basis

1. Example of basis in R^2
2. Example of a subset which is not basis in R^2
3. Example of basis in R^3
4. Example of a subset which is not basis in R^3

Practical 5 : Row space and column space

1. Find row space of 3 x 3 matrix
2. Find row space of 3 x 5 matrix
3. Find row space of 5 x 3 matrix
4. Find row space and column space of 3 x 3 matrix and compare them.
5. Find row space and column space of 3 x 5 matrix and compare them.

Practical 6 : Linear transformations

1. Show $T: R^2 \to R^2$ is a linear transformation.
2. Show $T: R^2 \to R^3$ is a linear transformation.
3. Show $T: R^3 \to R^3$ is a linear transformation.
4. Check whether $T: R^2 \to R^2$ is a linear transformation.
5. Check whether $T: R^2 \to R^3$ is a linear transformation.

Practical 7 : Kernel and image

1. Find the kernel space of a linear transformation $T: R^2 \to R^2$
2. Find the basis of kernel space of a linear transformation $T: R^4 \to R^3$
3. Find the range space of a linear transformation $T: R^2 \to R^3$
4. Find the range space of a linear transformation $T: R^3 \to R^2$
5. Verify Rank Nullity theorem.

Practical 8 : Eigenvalues and eigenvectors of 2 x 2 matrix

1. Find eigenvalues of 2 x 2 matrix with distinct real eigenvalues
2. Find eigenvalues of 2 x 2 matrix with repeated real eigenvalues
3. Find eigenvalues of 2 x 2 matrix with complex eigenvalues
4. Find the characteristic polynomial and largest eigenvalue.

Practical 9 : Eigenvalues and eigenvectors of 3 x 3 matrix

5. Find eigenvalues of 3 x 3 matrix with distinct real eigenvalues
6. Find eigenvalues of 3 x 3 matrix with repeated real eigenvalues
7. Find eigenvalues of 3 x 3 matrix with complex eigenvalues
8. Find the characteristic polynomial and largest eigenvalue.

Practical 10 : Diagonalization

1. Diagonalisation of a 2 x 2 matrix
2. Diagonalisation of a 3 x 3 matrix
3. Find $P^{-1}AP$ for a given matrix of size 2x2 matrix.
4. Find $P^{-1}AP$ for a given matrix of size 3x3 matrix.

Practical 11 : Cayley Hamilton theorem

1. Verification of Cayley Hamilton theorem of 2 x 2 matrix.
2. Verification of Cayley Hamilton theorem of 3 x 3 matrix.
3. Finding inverse of matrix using Cayley Hamilton theorem
4. Find power of a matrix using Cayley Hamilton theorem.

Practical 12 : Inner product spaces

- Show that given product is an inner product space
- Show that given product is an inner product space
- Check whether the given product is an inner product space.
- Problem based on computing inner product and angle between two vectors in V

Practical 13 : Orthogonal and orthonormal sets.

1. Problem based on finding orthogonal projection of a vector along another vector.
2. Problem based on finding orthogonal projection of a vector along a subspace
3. Problem based on checking orthogonal sets.
4. Problem based on orthonormal sets.
5. Problem on converting orthogonal set to orthonormal set.

Practical 14 : Gram-Schmidt process in R^2

1. Finding orthogonal projection of one vector on span of other vector
2. Converting a linearly independent set to an orthogonal set.
3. Applying Gram Schmidt on two vectors in R^2 (example 1)
4. Applying Gram Schmidt on two vectors in R^2 (example 2)

5. Applying Gram Schmidt on two polynomials in P_2

Practical 15 : Gram-Schmidt process in R^3

1. Finding orthogonal projection of one vector on span of two vector
2. Converting a linearly independent set to an orthogonal set.
6. Applying Gram Schmidt on two vectors in R^3 (example 1)
7. Applying Gram Schmidt on two vectors in R^3 (example 2)
8. Applying Gram Schmidt on three polynomials in P_2 (P_2 - the vector space of polynomials with degree ≤ 2)

Reference Books :

1. Schaum's outline of Theory and Problems of Linear Algebra by Seymour Lipschitz and Marc Lars Lipson, 3rd edition, 1968.
2. Introduction to Linear Algebra by Gilbert Strang, Wellesley-Cambridge Press,U.S.; 6th edition,2023.
3. Matrix and Linear Algebra aided with Matlab by Kanti Bhushan Datta, PHI Learning Pvt. Ltd., New Delhi, Second edition, 2012.
4. Linear Algebra, A geometric approach by S. Kumaresan, PHI Learning Pvt. Ltd., New Delhi, 2009.
5. Linear Algebra and its Applications by David C. Lay, Steven Lay, Judy McDonald Sixth Edition, 2022.
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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4401
Course Name: Basics of Operations Research
Teaching Scheme: TH: 2 Hours/Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Prerequisites: Basic knowledge of linear equations, linear inequality, graphs.

Course Objectives: The aim of this course is to study

- How to formulate real world problems into mathematical models.
- How to find optimum solution of real-world problems using various techniques of operations research.
- Scope and applications of operations research.
- How to build capabilities in the students for analysing different situations in the industrial scenario involving limited resources and finding the optimal solution within constraints.

Course Outcomes: On completion of the course, student will able to

- Apply the concept of operations research in solving complex problems in economics, finance, defense and management science.
- Design the real-life problems which help in decision making.
- Find optimum solution of formulated linear programming problem.
- Solve assignment problems.
- Formulate and solve two-person zero sum games.
- Construct various mathematical models in operations research.

Course Contents:

Unit 1	**Introduction to operations research**	**05 Lectures**
	• History of operations research • Objective of History of operations research • Applications of operations research	

	• Types of mathematical models in operations research • Limitations of operations research	
Unit 2	**Linear programming problems**	**13 Lectures**
	• Formulation of linear programming problem • Advantages of linear programming problem • Solutions of linear programming problem using graphical method • Assignment problem • Solution of assignment problem	
Unit 3	**Game theory**	**12 Lectures**
	• Formulation of games • Two person-zero sum game • Games with and without saddle point • Graphical solution (2x n, m x 2 game)	
	Total Lectures	**30 Lectures**

Reference Books:

1. Operations Research (Techniques for Management) by V.K. Kapoor, S. Chand, fifth Edition, 2001.
2. Operations Research (Theory and Applications) by J.K. Sharma, Macmilan India Ltd., fifth Edition, 2013.
3. Operations Research (Theory, Methods & Applications) by S.D. Sharma, Kedarnath Ramnath & co, 1992.

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Second Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU4601
Course Name: Lab Course on Scilab

Teaching Scheme: PR: 4 Hours/Week **Credit: 02**
Examination Scheme: CIA: 20 Marks **End-Sem: 30 Marks**

Course Objectives: To study Scilab to

- Find greatest common divisor and least common multiple of given two integers
- Define a polynomial and find its roots
- Represent the Matrix and perform operations on matrices
- Solve the system of linear equations
- Evaluate eigenvalues and eigenvectors of matrices.
- Draw the graphs of functions
- Write a program for numerical methods.
- Study special functions.
- Solve problems of Differential Calculus

Course Outcomes: On completion of the course, Using Scilab, student will be able to: -

- Find GCD and LCM of given two integers
- Define a polynomial and find its roots
- Declare the matrices and perform operations on them
- Solve system of linear equations in Scilab
- Find eigenvalues and eigenvectors of matrix
- Plot graph for one variable and two variables functions.
- Write a program for numerical methods
- Solve problems of Differential Calculus

Course Contents:

	List of Practicals	**60 Hours**
Practical 1	Introduction of Scilab	
Practical 2	GCD, LCM and Primes	
Practical 3	Trigonometry	
Practical 4	Polynomials	
Practical 5	Representation of Matrices	
Practical 6	Operations on Matrices	
Practical 7	System of linear equations	
Practical 8	Eigenvalues and eigenvectors	
Practical 9	Graphs of functions of one variable	
Practical 10	Graphs of functions of two variables	
Practical 11	Numerical methods for finding roots of equations	
Practical 12	Numerical interpolation	
Practical 13	Numerical integration	
Practical 14	Special Functions	
Practical 15	Differential Calculus	

Reference Books:

a. Scilab: A free software to MATLAB, by Achuthsankar S Nair, S, Chand and Company LTD. Company, 2012.
b. Introduction to Scilab: For Engineers and Scientists, by Sandeep Nagar, Apress, 2017.
c. Computing in Scilab, by Chetana Jain, Cambridge University Press, 2023.
d. Modeling and Simulations in Scilab/Scicos by Stephen Campbell, Jean Philippe Chancellor, Ramine Nikoukhah, Springer, 2006.
e. https://www.ee.iitm.ac.in/~hsr/scilab/manual.pdf
f. https://www.scilab.org/sites/default/files/Scilab_beginners.pdf

Level:- 5.5 (Third Year) Sem:V

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Third Year of B.Sc.
(2024 Course under NEP 2020)

Course Code: 24ScMatU5101
Course Name: Real Analysis
Teaching Scheme: TH: 4 Lectures/Week **Credit : 4**
Examination Scheme: CIA: 40 Marks **End-Sem: 60 Marks**

Prerequisites: Concepts of Differential Calculus, Integral Calculus
Course Objectives: The aim of this course is to study

- To develop understanding of the real number system and the concepts of limits and sequences.
- To study the behavior of sequences and series, including convergence, divergence, and the various tests used to determine convergence.
- To understand the concept of Riemann integral, fundamental theorems.
- To study the behavior of sequences and series of functions, including convergence, divergence, and the various tests used to determine convergence.

Course Outcomes: On completion of the course, student will able to understand

- Students will be able to prove the theorems and concepts in real analysis.
- Students will be able to apply various convergence tests to sequences and series.
- Students will develop the ability to apply Riemann integral for solving problems.
- Students will gain the ability to examine the convergence of function series, understanding the differences between pointwise and uniform convergence.

Course Contents:

Unit 1	**Sequences of Real Numbers**	**12 Lectures**
	• Definition of sequence and subsequence • Convergent sequences • Divergent sequences • Limit superior, Limit inferior • Cauchy sequences	
Unit 2	**Series of Real numbers**	**12 Lectures**
	• Convergent and divergent series • Series with non-negative terms • Alternating series, Conditional and Absolute convergence • Tests of absolute convergence	
Unit 3	**Riemann Integral**	**14 Lectures**

	• Definition and existence of Riemann integral • Properties of Riemann integral • Fundamental theorem of integral calculus • Improper Integrals	
Unit 4	**Sequences and series of functions**	**10 Lectures**
	• Point wise and uniform convergence of sequences of functions • Convergence and uniform convergence of series of functions • Weierstrass's M-Test for uniform convergence of series of functions	
	Total Lectures	**60 Lectures**

Text/Reference Books:

1. R. R. Goldberg, Methods of real analysis, Oxford & I. B. H. Publications, 1970.
2. First course in mathematical analysis, D somasundaram, B Chaudhari, Narosa Publishing house 2009.
3. Shanti Narayan and Mittal - A course of Mathematical Analysis, Revised edition, S. Chand and Co.(2002).
4. Ajit Kumar and S.Kumaresan, A Basic Course in Real Analysis, CRC Press, Second Indian Reprint 2015.
5. S.C. Malik and Savita Arora - Mathematical Analysis , New Age International Publications,Third Edition,(2008).
6. Robert, G. Bartle, Donald Sherbert - Introduction to real analysis, Third edition, John Wiley and Sons.