### Modern College of Arts, Science and Commerce,

Shivajinagar, Pune – 5

Second Year of B.Sc. 2023

(Course under NEP 2020)

Course Code: 23ScStaU3101

Course Name: Discrete Probability distributions and Time series.

Teaching Scheme: 4 Hours/Week Credit: 4

Examination Scheme: CIA: 40 Marks End-Sem: 60 Marks

**Course Outcomes:** On completion of the course, student will be able to:

1. Distinguish between standard probability distribution and truncated distribution.

- 2. Identify real life situations where multinomial distributions can be applied.
- 3. Fit Time series models.
- 4. Forecast time series values using various methods of time series.
- 5. Differentiate between methods of estimating trend.
- 6. Differentiate between methods of estimating seasonal variation.

Unit 1	Bivariate discrete probability distribution	14
		Lectures
	1.1 Definition of two-dimensional discrete random	
	variable, its joint p.m.f. and its distribution function and their	
	properties, concept of identically distributed random variables.	
	1.2 Computation of probabilities of events in bivariate probability	
	distribution.	
	1.3 Concepts of marginal and conditional probability distributions.	
	1.4 Independence of two discrete random variables based on joint and	
	marginal p.m.f.	
	1.5 Definition of raw and central moments.	

	1.6 Theorems on expectations of sum and product of two jointly	
	distributed random variables.	
	1.7 Conditional expectation.	
	1.8 Definitions of conditional mean and conditional variance.	
	1.9 Definition of covariance, coefficient of correlation,	
	1.10 Variance of linear combination of variables	
	1.11 Illustrations of some standard bivariate probability distributions	
	1.12 Conditional distribution of X given (X+Y) for binomial distribution.	
	1.13 Examples and problem	
Unit 2	Multinomial Distribution	14 Lectures
	2.1 Probability mass function (p. m. f.)	
	$P(X_1 = x_1, X_2 = x_2,, X_k = x_k) = \frac{n! p_1^{x_1} p_2^{x_2} p_k^{x_k}}{x_1! x_2! x_k!}$	
	$x_i = 0, 1, 2,, n - \sum_{1}^{i-1} x_r,$	
	$i = 1, 2, \dots, k$	
	$0 < p_i < 1; i = 1, 2,, k;$	
	$p_1 + p_2 + \dots + p_k = 1;$	
	Notation: $(X_1, X_2,, X_k) \sim MD(n, p_1, p_2,, p_k)$ ,	
	$\underline{X} \sim MD(n, \underline{p}),$	
	where $\underline{X} = (X_1, X_2,, X_k), \underline{p} = (p_1, p_2,, p_k).$	
	2.2 Joint MGF of $(X_1, X_2,, X_k)$ , use of MGF to obtain means, variances,	
	covariances, total correlation coefficients, multiple and partial	
	correlation coefficients for $k=3$ .	
	2.3 univariate marginal distribution, distribution of $X_i + X_j$ ,	
	2.4 conditional distribution of $X_i$ given $X_i + X_j = r$ ,	
	2.5 variance- covariance matrix and its interpretation and real life situations	
	and applications.	
Unit 3	Truncated Distributions:	6 Lectures
	3.1 Concept of truncated distribution, truncation to the right, left and on both sides.	

	3.2	Binomial distribution left truncated at $X = 0$ (value zero is	
		discarded), its p.m.f., mean and variance.	
	3.3	Poisson distribution left truncated at $X = 0$ (value zero is discarded),	
		its p.m.f., mean and variance.	
	3.4	Real life situations and applications.	
Unit 4	]	Time Series	26 lectures
	4.1	Meaning and utility of time series, components of time series: trend,	
		seasonal variations, cyclical variations, irregular (error) fluctuations,	
		Autocovariance function (ACVF) and Auto correlation function	
		(ACF), PACF, Introduction of stationarity and non-stationarity,	
	,	white-noice	
	4.2	Exploratory data analysis: Time series plot to	
		(i) check any trend & seasonality in the time series	
		(ii) capture trend.	
	4.3	Methods of trend estimation and smoothing:	
		(i) moving average, (ii) curve fitting by least square	
		principle, (iii) exponential smoothing.	
	4.4	Choosing parameters for smoothing and forecasting.	
	4.5	Forecasting based on exponential smoothing.	
	4.6	Measurement of seasonal variations: i) simple average method, ii)	
		ratio to moving average method, iii) ratio to trend where linear trend	
		is calculated by method of least squares.(To be taken in practical)	
	4.7	Fitting of autoregressive model AR (p), p=1,2, Moving	
		average model(MA(q), q= 1,2), Plotting of residuals.	
	4.8	Data Analysis of Real Life Time Series:	
		Price index series, share price series, economic time series,	
		sales tax series, market price of daily consumables, weather related	
		time series: temperature and rainfall time series, wind speed	
		time series, pollution levels.	
		time series, pollution levels.	

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- 2. Chatfield C. (2001), The Analysis of Time Series An Introduction, Chapman and Hall / CRC, Texts in Statistical Science.
- 3. Dixit P.G., Kapre P.S., Pawgi V. R. Discrete Probability Distribution, Time Series And R Software, Nirali Prakashan.
- 4. Goon A. M., Gupta, M. K. and Dasgupta, B. (1986), Fundamentals of Statistics, Vol. 2, World Press, Kolkata.
- 5. Gupta, S. C. and Kapoor, V. K. (2002), Fundamentals of Mathematical Statistics, Eleventh Edition), Sultan Chand and Sons, 23, Daryaganj, New Delhi, 110002.
- 6. Gupta, S. C. and Kapoor V. K. (2007), Fundamentals of Applied Statistics (Fourth Edition), Sultan Chand and Sons, New Delhi.
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- 8. Meyer, P. L., Introductory Probability and Statistical Applications, Oxford and IBH Publishing Co. New Delhi.
- 9. Mood, A. M., Graybill F. A. and Bose, F. A. (1974), Introduction to Theory of Statistics McGraw Hill Series G A 276
- 10. Mukhopadhya Parimal (1999), Applied Statistics, New Central Book Agency, Pvt. Ltd. Kolkata
- 11. Purohit S. G., Gore S. D. and Deshmukh S. R. (2008), Statistics using R, Narosa Publishing House, New Delhi.
- 12. Ross, S. (2003), A first course in probability (Sixth Edition), Pearson Education publishers, Delhi, India.
- 13. Walpole R. E., Myers R. H. and Myers S. L. (1985), Probability and Statistics for Engineers and Scientists, Macmillan Publishing Co. Inc. 866, Third Avenue, New York 10022.
- 14. Weiss N., Introductory Statistics, Pearson education publishers..

## Progressive Education Society's Modern College of Arts, Science and Commerce, Shivajinagar, Pune – 5 Second Year of B.Sc. 2023 (Course under NEP 2020)

Course Code: 23ScStaU3102

**Course Name: Statistics Practical III** 

Teaching Scheme4 Hours/Week Credit: 2

Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks

**Course Outcomes:** On completion of the course, student will be able to:

- 1. Distinguish between standard probability distribution and truncated distribution.
- 2. Identify real life situations where multinomial distributions can be applied.
- 3. Fit Time series models
- 4. Forecast time series values using various methods of time series.
- 5. Differentiate between methods of estimating trend.
- 6. Differentiate between methods of estimating seasonal variation.

### **Practicals:**

Sr. No.	Title of the practical	No. of
		practicals
1	Estimation and elimination of trend component using least	1
	square method, Moving average method, exponential	
	smoothing method	

2	Estimation and elimination of seasonal component using	1
	ratio to trend and ratio to moving average method	
3	Estimation and elimination of trend and seasonal	1
	component	
4	Modeling of time series I	1
5	Modeling of time series II	1
6	Modeling of time series III	1
7	Bivariate Probability distributions	1
8	Applications of Multinomial probability distribution	1
9	Applications of Truncated probability distribution	1
10	Queuing models I	1
11	Queuing models II	1
12	Queuing models III	1
13-15	Experiential learning	3

## Progressive Education Society's Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune – 5 Second Year of B. Sc. Statistics (NEP) Minor

Course Code: 23ScStaU3301 (Theory)

Course Name: Introduction to Probability

Teaching Scheme: 2 Lectures / Week Credit: 02

**Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks** 

**Course Outcomes**: At the end of the course students will be able to:

- 1. differentiate between problems based on multiplication principle and addition principle.
- 2. Distinguish between Permutations and combinations
- 3. Study the concept of types of events
- 4. Study the concept of probability of event
- 5. Study the independence of an events
- 6. Understand the concepts of conditional probability and Bayes' theorem

Unit	Title and Contents	No. of Lectures
1	Permutations and combinations	7

	1.1 Factorial notation	
	1.2 Fundamental of principle of counting Principle of addition and	
	principle of multiplication	
	1.3 Concept of permutation Permutation when all the objects are	
	distinct Permutation when not all the objects are distinct	
	1.4 Concept of Combination	
	1.5 Examples and Problem.	
	1.6	
2	Concepts of Events	6
	2.1 Experiments/Models, deterministic and non- deterministic	
	models. Random experiment, concept of statistical regularity.	
	2.2 Definitions of - (i) Sample space, (ii) Discrete sample space:	
	finite and countably infinite, (iii) Continuous sample space,	
	(iv) Event, (v) Elementary event, (vi) Complement of an event, (vii)	
	Certain event, (viii) Impossible event.	
	2.3 Concept of occurrence of an event.	
	2.4 Algebra of events and its representation in set theory notation.	
3	Introduction of Probability	6
	3.1 Classical definition of probability and its limitations.	
	3.2 Probability model, probability of an event, equiprobable and	
	non-equiprobable sample space	
	3.3 Axiomatic definition of probability, theorems on probability,	
	proofs based on axiomatic definition, computation of	
	probability of an event.	
4	Independence of events, Conditional Probability	7
	4.1 Definition of independence of two events	
	4.2 Pairwise independence and mutual independence for three events	
	variance, real life situations, comments on mode and median.	
	4.3 Definition of conditional probability of events.	
	4.4 Multiplication therom $P(A \cap B) = P(A) * P(B A)$	
	Generalization of $P(A \cap B \cap C)$	
5	Bayes' Theorem	4

- 5.1 Partition of the sample space
- 5.2 Proof of Bayes' theorem. Applications of Bayes' theorem in real life.

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### **References:**

- 1. Dr. Dixit P.G, Dr. Prayag V.R., Kapre P. S. Discrete probability and Probability distributions, Nirali Prakashan, Pune
- 2. Gupta, S. C. and Kapoor, V. K. (1997). Fundamentals of Applied Statistics, 3<sup>rd</sup> Edition, Sultan Chand and Sons Publishers, NewDelhi.
- 3. Hogg,R.V.andCraigR.G.(1989).IntroductiontoMathematicalStatistics, MacMillan Publishing Co., NewYork.
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- 6. Hoel P. G. (1971). Introduction to Mathematical Statistics, John Wiley and Sons, NewYork.
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- 8. Ross S. (2002). A First Course in Probability, 6<sup>th</sup> edition, Pearson Education, Inc. & Dorling Kindersley Publishing,Inc.
- 9. *Roussas*, George G. (2016). First course in mathematical *statistics*. 2<sup>nd</sup>edition *Publisher*: AcademicPress.
- 10. Walpole and Myres, (1986). Mathematical Statistics, 4<sup>th</sup> edition, Publisher: Longman Higher Education.

<u>Links:</u> https://mahades. mamarashtra.gov.in

### www.mospi.gov.in

http://www.isical.ac.in https://iipsindia.org

## Progressive Education Society's Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune – 5 Second Year of B. Sc. Statistics (NEP) Minor

Course Code: 23ScStaU3301 (Practical) Course Name: Introduction to

**Probability** 

Teaching Scheme: 4 hours /Week Credit: 02

**Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks** 

**Pre-requisites**: Knowledge of the topics in theory papers II

Course Outcome: At the end of this course students will be able to:

1. To apply permutations on various situations.

- 2. To deal with problems where order of arrangement plays important role.
- 3. To compute probabilities.
- 4. To use theorems of probabilities.
- 5. To compute conditional probabilities
- 6. To apply Bayes' theorem.

	Title of Experiment/ Practical
1	Permutations and combinations I
2	Permutations and combinations II
3	Permutations and combinations III
4	Permutations and combinations IV
5	Problems based on probabilities -I
6	Problems based on probabilities -II
7	Problems based on probabilities -III
8	Problems based on probabilities -IV
9	Problems based on probabilities -V
10	Problems based on probabilities -VI
11	Problems based on probabilities -VII
12	Problems based on probabilities –VIII
13	Project equivalent to 3 practical's

### Modern College of Arts, Science and Commerce,

### Shivajinagar, Pune – 05

### Second Year B.Sc. (Course under NEP 2020) (OE)

Course Code: 23ScStaU3401 Course Name: Applied Statistics - I

Teaching Scheme: 2 hours/week Credits: 02

Examination Scheme: CIA: 20 Marks End Sem: 30 Marks

**Course Outcomes:** At the end of this course students will be able to:

- 1. Understanding and Application of Regression Analysis. Evaluation and Interpretation of Regression Models:
- 2. Identification and Remediation of Multicollinearity. Diagnostic Checks and Assumption Validation:
- 3. Understanding of Operations Research Concepts:
- 4. Application of Operations Research Techniques. Solution of Linear Programming Techniques, Transportation and Assignment Problems:
- 5. Comprehension of Index Number Concepts. Types and Construction Techniques:
- 6. Uses and Applications of Index Numbers. Practical Problem-solving with Index Numbers:

Unit 1	Probability and Applications of Probability	06 lectures
	Introduction of Probability	
	1.1 Classical definition of probability and its limitations.	
	1.2 probability of an event, equiprobable and non-equiprobable	
	sample space	
	1.3 Axiomatic definition of probability,	
	1.4 computation of simple probabilities, conditional Probabilities,	
	probabilities based on independence events.	
Unit 2	Applications of probability distributions	06
		lectures
	Introduction to Bernoulli, Binomial and Poisson distribution and it's	
	applications.	
Unit 3	Operation Research	14
		lectures
	3.1 Introduction: Definition. Features of Operations Research Approach	
	Interdisciplinary Approach.	
	3.2 Formulations of problem as linear programming problem.	

	<ul> <li>3.3 Graphical method of finding optimal solution to linear programming problem.</li> <li>3.4 Assignment Problems: Introduction, finding optimal solution to Assignment problem by using Hungarian method.</li> <li>3.5 Replacement Models and its applications.</li> </ul>	
Unit 4	Index Numbers	04 lectures
	4.1 Introduction: Meaning and definition, Problems or Consideration in	
	the construction of Index numbers.	
	4.2 Types and Construction of Index Numbers: Simple and weighted.	
	4.3 Some specific Index Numbers (Laspeyre's, Paasche's, Fisher's Index	
	Numbers)	

- 1. Discrete probability and probability distributions-1 (F.Y.B.Sc./B.A.) by Dr. (Mrs.) V. R. Prayag, Dr. P.G.Dixit, P.S.Kapre, Nirali Publication.
- 2. Fundamentals of Mathematical Statistics, by Gupta and Kapoor, Sultan Chand and Sons, New Delhi.
- 3. Operation Research for Managerial Decision-making, by V. K. Kapoor, Sultan Chand and Sons, New Delhi.
- 4. Problems and solutions in Operation Research, by V. K. Kapoor, Sultan Chand and Sons, New Delhi.
- Descriptive Statistics-II (B.Sc. Part-I, SEM-II) by Prof. S.J. Alandkar, Dr. P.G.Dixit, Prof. N.
   L. Dhanshetti, Nirali Publication.
- 6. Fundamentals of Applied Statistics, by Gupta and Kapoor, Sultan Chand and Sons, New Delhi.

### Modern College of Arts, Science and Commerce, Shivajinagar, Pune - 5 Second Year of B.Sc. (2023 Course under NEP 2020)

Course Code: 23StaU3501 Course Name: C-Programming

Teaching Scheme: 4 Hours/Week Credit: 02

**Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks** 

**Course Outcomes**: At the end of this course students will be able to:

1. Read and print the different types of data.

- 2. Carry out simple arithmetic calculations and find the value of functions of a variable.
- 3. Apply different control structures of C.
- 4. Apply Arrays in C.
- 5. Compute different Measures of central tendencies and dispersions.
- 6. Fit a simple linear regression model.
- 7. Fit a binomial model to a given data.
- 8. Apply different string operations.

Sr. No.	Title of experiment	No. of Practical
1	(i) Enter and print the different types of data types.	1
	(ii) To carry out arithmetic calculations.	
	(iii) Converting degree Celsius temperature to Fahrenheit.	
2	(i) To check given number is odd or even.	1
	(ii) To check given number is prime or not.	
3	(i) To find maximum number among three numbers	1
	ii) To check whether a given number 'm' is divisible by 'n' or not.	
4	(i) To find area of triangle using Herons Formula.	1
	(ii) To find area of circle.	
	(iii) To print Fibonacci series	

5	(i) To find roots of quadratic equation	1
	(ii) Newton-Raphson method to solve transcendental equation	
6	(i) To find sum of digits of a number.	1
	(ii) To find factorial of a given integer number.	
7	(i) To find value of X <sup>n</sup> where 'n' is integer.	1
	ii) To find GCD of two integer numbers.	
8	To evaluate $\exp(x)$ , $\sin(x)$ , $\log(x)$ etc. by Taylor's Series Expansion.	1
9	(i) To solve simultaneous linear equations. (two equations in two	1
	variables)	
	(ii) To convert decimal number to equivalent binary number	
10	(i) To find mean, geometric mean and Harmonic mean of 'n' numbers.	1
	(ii) To arrange the data in increasing/decreasing order of magnitude.	
	(iii) To find quartiles and mode of 'n' numbers.	
11	(i) To find variance, coefficient of variation, mean deviation from	1
	different measure of central tendency for given 'n' observations.	
	(ii) To prepare multiplication table.	
12	(i) To prepare a frequency distribution with given class interval.	1
	(ii) To find mean, quartiles, variance, coefficient of variation	
	for frequency distribution.	
13	(i) To find Correlation coefficient for a given bivariate data.	1
	(ii) To Fit regression line for a given bivariate data.	
14	(i) To test Palindrome string using string function	1
	(ii) To sort a string using string function.	
	(iii)To search string using string function.	
	(iv)To combine two strings using string function	
15	Experiential learning	1

- 1. Kanitkar Y (2008): Let us C, BFB publishers, New Delhi.
- 2. Rajaraman V. (2007): Computer Programming in C, Prentice Hall of India.
- 3. Peter van der Linden (1994.) :Expert C Programming: Deep C Secrets.

# Progressive Education Society's Modern College of Arts, Science and Commerce, Shivajinagar, Pune - 5 Second Year of B.Sc. (2023 Course under NEP 2020)

Course Code: 23ScStaU4101 Course Name: Continuous Probability Distributions

Teaching Scheme: 4 Hours/Week Credit: 04

Examination Scheme: CIA: 40 Marks End-Sem: 60 Marks

**Course Outcomes:** On completion of the course, student will be able to:

1. Compute the probability of various continuous probability distributions.

- 2. Study the probability distributions by studying the nature of probability curves.
- 3. Identify appropriate continuous probability distribution to real life situations.
- 4. Fit appropriate continuous probability distribution to real life situations.
- 5. Model sample from various continuous probability distributions.
- 6. Interrelate relation among the distributions.

Unit No.	Unit Name	Number of
		Lectures
Unit 1	Continuous Univariate Distributions:	10

	1.1 Continuous sample space: Definition, illustrations. Continuous random	
	variable: Definition, probability density function (p.d.f.), cumulative	
	distribution function (c.d.f.), properties of c.d.f.(without proof),	
	probabilities of events related to random variable.	
	<b>1.2</b> Expectation of continuous r.v., expectation of function of r.v.	
	E[g(X)], mean, variance, geometric mean, harmonic mean, raw and	
	central moments, skewness, kurtosis, mean deviation about mean.	
	<b>1.3</b> Moment generating function (MGF): Definition, properties. Cumulant	
	generating function (CGF): Definition.	
	<b>1.4</b> Mode, partition values : quartiles( $Q_1, Q_2, Q_3$ ), deciles, percentiles.	
	<b>1.5</b> Probability distribution of function of r. v. : $Y = g(X)$ using	
	i) Jacobian of transformation for $g(.)$ monotonic	
	function and one-to-one, on to functions,	
	ii) Distribution function for $Y = X^2$ , $Y =  X $ etc.	
	iii) M.G.F. of $g(X)$ .	
Unit 2	Uniform or Rectangular Distribution:	03
Unit 2		03
Unit 2	Probability density function (p.d.f.)	03
Unit 2	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \end{cases}$	03
Unit 2	Probability density function (p.d.f.)	03
Unit 2	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ .	03
Unit 2	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ .  p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.	03
Unit 2	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ . $p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.$ Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the	03
Unit 2 Unit 3	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ . $p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.$ Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the c. d. f. of continuous r.v. $X$ . Application of the result to model sampling.	03 12
	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ . $p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.$ Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the c. d. f. of continuous r.v. $X$ . Application of the result to model sampling. (Distributions of $X + Y, X - Y, XY$ and $X/Y$ are not expected.)	
	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ .  p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.  Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the c. d. f. of continuous r.v. $X$ . Application of the result to model sampling. (Distributions of $X + Y, X - Y, XY$ and $X/Y$ are not expected.)  Normal Distribution:  Probability density function (p. d. f.)	
	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ .  p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.  Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the c. d. f. of continuous r.v. $X$ . Application of the result to model sampling. (Distributions of $X + Y, X - Y, XY$ and $X/Y$ are not expected.)  Normal Distribution:	
	Probability density function (p.d.f.) $f(x) = \begin{cases} \frac{1}{b-a}, & a \le x \le b \\ 0, & otherwise \end{cases}$ Notation: $X \sim U[a, b]$ .  p. d. f., sketch of p. d. f., c. d. f., mean, variance, symmetry, MGF.  Distribution of i) $\frac{X-a}{b-a}$ , ii) $\frac{b-X}{b-a}$ , iii) $Y = F(X)$ , where $F(X)$ is the c. d. f. of continuous r.v. $X$ . Application of the result to model sampling. (Distributions of $X + Y, X - Y, XY$ and $X/Y$ are not expected.)  Normal Distribution:  Probability density function (p. d. f.)	

	of probability curve, mean, variance, (MGF, CGF, central moments,	
	cumulants, skewness, kurtosis, mode, quartiles $(Q_1,Q_2,Q_3)$ (statement	
	only) ), points of inflexion of probability curve, mean deviation,	
	additive property, probability distribution of:	
	i) $\frac{X-\mu}{\sigma}$ , standard normal variable (S.N.V.),	
	ii) $aX + b$ ,	
	iii) $aX + bY + c$ , where X and Y are independent normal variates.	
	Probability distribution of $\bar{X}$ , the mean of n i.i.d. N $(\mu, \sigma^2)$ r.v s.,	
	computations of normal probabilities using normal probability	
	integral tables. Central limit theorem (CLT) for i.i.d. r.v.s. with	
	finite variance (statement only), its illustration for Poisson and	
	Binomial distributions.	
Unit 4	Exponential Distribution:	05
	Probability density function (p. d. f.)	
	$f(x) = \begin{cases} \alpha e^{-\alpha x} ; & x \ge 0, \alpha > 0 \\ 0 & ; otherwise \end{cases}$	
	Notation : $X \sim Exp(\alpha)$ .	
	Nature of density curve, interpretation of $\alpha$ as a scale and $\frac{1}{\alpha}$ as mean,	
	mean, variance, MGF, CGF, skewness, kurtosis, c.d.f., graph of c.d.f.,	
	lack of memory property, quartiles $(Q_{1,}Q_{2,}Q_{3})$ , mean deviation about	
	mean, distribution of sum of k i.i.d exponential random variables.	
Unit 5	Gamma Distribution:	06
		00
	Probability density function (p. d. f.)	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$ Notation: $X \sim G(\alpha, \lambda)$ ,	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$ Notation: $X \sim G(\alpha, \lambda)$ ,  Nature of probability curve, special cases: i) $\alpha = 1$ , ii) $\lambda = 1$ ,	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$ Notation: $X \sim G(\alpha, \lambda)$ ,  Nature of probability curve, special cases: i) $\alpha = 1$ , ii) $\lambda = 1$ ,  MGF, CGF, moments, cumulants, skewness, kurtosis, mode,	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$ Notation: $X \sim G(\alpha, \lambda)$ ,  Nature of probability curve, special cases: i) $\alpha = 1$ , ii) $\lambda = 1$ ,  MGF, CGF, moments, cumulants, skewness, kurtosis, mode, additive property. Distribution of sum of $n$ i.i.d. exponential	
	$f(x) = \begin{cases} \frac{\alpha\lambda}{\Gamma\lambda} x^{\lambda-1} e^{-\alpha x} & ; x > 0, \ \alpha, \lambda > 0 \\ 0 & ; \ otherwise \end{cases}$ Notation: $X \sim G(\alpha, \lambda)$ ,  Nature of probability curve, special cases: i) $\alpha = 1$ , ii) $\lambda = 1$ ,  MGF, CGF, moments, cumulants, skewness, kurtosis, mode, additive property. Distribution of sum of $n$ i.i.d. exponential variables. Relation between distribution function of Poisson and	

Unit 6	Chi-square Distribution:	08
	Definition as a sum of squares of i.i.d. standard normal variables.  Derivation of the p.d.f. of Chi-square variable with n degrees of freedom (d.f.) using MGF.	
	Notation: $X \sim \chi_n^2$ Mean, variance, MGF, CGF, central moments skewness, kurtosis, mode, additive property. Use of chi-square tables for calculations of probabilities.  Normal approximation: $\frac{\chi_n^2 - n}{\sqrt{2n}}$ (statement only)	
Unit 7	Student's t -distribution:	08
	Definition of $t$ r.v. with n d.f. in the form of $=\frac{U}{\sqrt{\frac{V}{n}}}$ , where $U \sim N$ (0,1) and $V$ is chi-square with n d.f., where $U \& V$ are independent random variables. Notation: $t \sim t_n$	

Unit 8	Snedecore's F –distribution:	08
	Definition of $F$ r.v. with $n_1$ and $n_2$ d.f. as $F_{n_1,n_2} = \frac{X_1/n_1}{X_2/n_2}$ where $X_1 \& X_2$	
	are independent chi-square variables with $n_1$ and $n_2$ d.f.	
	Notation: $F \sim F_{n_1,n_2}$	
	Derivation of the p.d.f, nature of probability curve, mean, variance,	
	(moments (derivations are not expected)), mode.	
	Distribution of $\frac{1}{F_{n_1,n_2}}$ use of $F$ —tables for calculation of probabilities.	
	Interrelationship between Chi-square, $t$ and $F$ distributions.	
Reference	<u>es :</u>	
1. Dixit F	P.G., and Kapre P.S. Sampling distributions and Inference, Nirali Prakashan.	
2. Gupta	, S. C. and Kapoor, V. K. (2002), Fundamentals of Mathematical Statistics,	
Ele	venth Edition), Sultan Chand and Sons, 23, Daryaganj, New Delhi, 110002.	

- 3. Hogg, R. V. and Craig, A. T., Mckean J. W. (2012), Introduction to Mathematical Statistics (Tenth Impression), Pearson Prentice Hall.
- 4. Meyer, P. L., Introductory Probability and Statistical Applications, Oxford and IBH Publishing Co. New Delhi.
- 5. Mood, A. M., Graybill F. A. and Bose, F. A. (1974), Introduction to Theory of Statistics McGraw Hill Series G A 276
- 6. Mukhopadhya Parimal (1999), Applied Statistics, New Central Book Agency, Pvt. Ltd. Kolkata .
- 7. Purohit S. G., Gore S. D. and Deshmukh S. R. (2008), Statistics using R, Narosa Publishing House, New Delhi.
- 8. Ross, S. (2003), A first course in probability (Sixth Edition), Pearson Education publishers, Delhi, India.
- Walpole R. E., Myers R. H. and Myers S. L. (1985), Probability and Statistics for Engineers and Scientists, Macmillan Publishing Co. Inc. 866, Third Avenue, New York 10022

# Progressive Education Society's Modern College of Arts, Science and Commerce, Shivajinagar, Pune - 5 Second Year of B.Sc. (2023 Course under NEP 2020)

Course Code: 23ScStaU4102 Course Name: Statistics Practical IV

Teaching Scheme: 8 Hours/Week Credit: 04

**Examination Scheme: CIA: 40 Marks Course Outcomes:** On completion of the course, student will be able to:

- 1. Do matrix algebra; find determinant and inverse of matrix.
- 2. Solve simultaneous equations using matrix algebra.
- 3. Formulate the real life problem in Linear Programming Problem.
- 4. Find optimal solution to Linear Programming Problem.
- 5. Compute the probability of continuous probability distributions.
- 6. Identify appropriate continuous probability distribution to real life situations.
- 7. Fit appropriate continuous probability distribution to real life situations.
- 8. Model sample various continuous probability distribution.
- 9. Apply appropriate non-parametric test to a real life data sets.

	PRACTICALS Based on Linear algebra, Operation Research and Continuous Probability Distributions	
	Title of Experiment/ Practical	No. of practicals
1	Solving the system of linear equations	1
2	Matrices – I: (finding Determinant ,Rank , Inverse of matrix via adjoint matrix)	1
3	Matrices-II: (finding Basis, linearly dependent and independent vectors)	1
4	Finding Eigen values and Eigen vectors of matrices	1
5	Introduction to Operation Research (OR) and Linear programming problem (LPP), different applications of OR , Formulation of a given problem in a LPP .	2
6	Solving LPP using graphical method (Cases: feasible solution, infeasible solution, no solution, unbounded solution, alternate solution)	1
7	Solving LPP using simplex method (Cases: feasible solution, infeasible solution, unbounded solution, alternate solution)	2

8	Solving LPP using Big-M method.	1
9	Introduction to Transportation problem (TP) and finding initial basic feasible solution using least cost method, Vogel's approximation method.	1
10	Finding Optimal solution of TP using modified distribution method.	1
11	Solving an Assignment problem using Hungarian method	1
12	Applications of exponential distribution, Model sampling from exponential distribution	1
13	Model sampling from normal distribution using	1
	(i) distribution function	
	(ii) Box-Muller transformation.	
14	Applications of normal distribution.	1
15	Fitting of exponential distribution and computation of expected frequencies.	1
16	Fitting of normal distribution and computation of expected frequencies.	1
17	Testing of hypotheses: basic concepts: null hypothesis, alternative hypothesis, composite hypothesis, one sided and two sided alternatives, probability of type-I error, probability of Type II error, level of significance.	1
18	Tests based on normal distribution.	2
19	Tests based on chi-square distribution.	2
20	Tests based on t distribution.	2
21	Tests based on F distribution.	1
22	Mixed sets of problems on testing of hypotheses based on normal, chi-square, t and F distribution.	2
23- 30	Experiential learning	3

- 1. Linear Algebra and its Applications (5th Edition) David C Lay, Steven R. Lay, Judi J. Mac Donald Pearson Publication, Fifth Edition, 2016.
- 2. Elementary Linear Algebra with supplemental Applications, by Howard Anton and others, Wiley Student Edition, Fourth edition.
- 3. Gupta, P.K. and Hira, D.S.(2008). Operation Research, 3 rd edition S. Chand and company Ltd., New Delhi. 3.
- 4. Kapoor, V. K.(2006). Operations Research, S. Chand and Sons. New Delhi.
- 5. Dixit P.G., and Kapre P.S. Sampling distributions and Inference, Nirali Prakashan.
- 6. Gupta, S. C. and Kapoor, V. K. (2002), Fundamentals of Mathematical Statistics, Eleventh Edition), Sultan Chand and Sons, 23, Daryagani, New Delhi, 110002.
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- 11. Purohit S. G., Gore S. D. and Deshmukh S. R. (2008), Statistics using R, Narosa Publishing House, New Delhi.
- 12. Ross, S. (2003), A first course in probability (Sixth Edition), Pearson Education publishers, Delhi, India.

### Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune – 5

Second Year of B. Sc. Statistics (NEP) (Minor)

Course Code: 23ScStaU4301 (Theory) Course Name: Probability Distributions

Teaching Scheme: 2 Lectures / Week Credit: 02

Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks

**Course outcomes**: At the end of the course the students will be able to:

- 1. Compute probabilities for different standard discrete probability distributions.
- 2. Study the probability distributions by studying the nature of probability curves.
- 3. Identify different real life situations where standard discrete probability distributions can be applied.
- 4. Fit appropriate continuous probability distribution to real life situations.
- 5. Model sample from various discrete probability distributions.
- 6. Identify limiting Distributions.

Unit	Title and Contents	No. of Lectures
1	Univariate Probability Distributions	7
	1.1 Concept and definition of a discrete random variable.	
	1.2 Probability mass function (p.m.f.) and distribution function (d.f.), $F_X(\cdot)$ , of	
	discrete random variable, Characteristic properties of distribution function.	
	1.3 Mode and median of a univariate discrete probability distribution.	
2	Some standard discrete probability distributions based on finite sample space I	6
	2.1 Uniform discrete distributions on integers 1 to n $p.m.f P(X = x) = \begin{cases} 1/n & x = 1,2, \dots n \\ 0 & otherwise \end{cases}$ mean, variance, real life situation, comments on mode and median  2.2 Bernoulli Distributution: p.m.f. $P(X=x) = \begin{cases} p^{x}(1-p)^{1-x} & , x = 0,1 \\ 0 & otherwise \end{cases}$ Mean, variance, M.G.F. and moments. Distribution of sum of two independent Bernoulli random variable	

2.3 Binomial distribution: p.m.f.	
$P(X = x) = \begin{cases} \binom{n}{x} p^x q^{n-x} & x = 0,1,2, \dots n, ; 0$	
Notation : $X \sim B(n,p)$ ., situations where this distribution is applicable.	
mean, variance, recurrence relation for successive probabilities,	
computation of probabilities of different events, mode.	
3 Some standard probability distributions based on accountably infinite sample space I	7
<b>3.1 Poisson distribution</b> : Notation : $X \sim P(\lambda)$ .	
$P(X=x) = \frac{e^{-\lambda}\lambda^x}{x!}$ $x=0,1,2; \lambda > 0$	
=0 otherwise	
Situations where this distribution is applicable. Mean, variance and	
m.g.f, and c.g.f., moments usingc.g.f., skewness ,kurtosis, recurrence	
relation, conditional distribution $X \mid X + Y$ , additive property, Poisson	
distribution as a limiting form of binomial distribution	
3.2 Geometric distribution:	
Notation: $X \sim G(p)$ ,	
Geometric distribution on support (0, 1, 2),	
with p.m.f. $p(x) = pq^x$ , $x = 0, 1, 2, 0 ,$	
q = 1 - p.	
0 otherwise	
Geometric distribution on support (1,2,)	
with p.m.f. $p(x) = pq^{x-1}$ , $x = 1, 2$ $0 ,$	
q = 1 - p.	
0 otherwise	
distribution function, recurrence relation, situations where this	
distribution is applicable. mean, variance, m.g.f, c.g.f., moments	
lack of memory property. Fitting of geometric distribution.	

### 3.3 Negative Binomial distribution:

$$P(X = x) = \begin{cases} \binom{x+k-1}{x} p^k q^x & x = 0,1,2,...n, ; 0 0 \\ 0 & otherwise \end{cases}$$

Nature of p.m.f, Negative binomial distribution as waiting time distribution ,m.g.f, c.g.f, mean , variance, skewness, kurtosis,( recurrence relation between moments is not expected) Relation between Geometric and negative binomial distribution. Poisson approximation to negative binomial distribution. Real life situations

### 3.4 Normal distribution.

Probability density function (p. d. f.)

$$f(x) = \begin{cases} \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} ; -\infty < x < \infty, -\infty < \mu < \infty, \ \sigma > 0 \\ 0 ; otherwise \end{cases}$$

Notation:  $X \sim N (\mu \sigma^2)$ .

p. d. f. curve, identification of scale and location parameters, nature of probability curve, mean, variance. Properties of normal distributions:

i) 
$$\frac{X-\mu}{\sigma}$$
, standard normal variable (S.N.V.),

ii) 
$$aX + b$$
,

iii) aX + bY + c, where X and Y are independent normal variates, Applications.

- 1. Agarwal B. L. (2003). Programmed Statistics, 2<sup>nd</sup> edition, New Age International Publishers, NewDelhi.
- B.L.S. Prakasarao, (2008). A First Course in Probability and Statistics, World Scientific Publishing Company
- 3. Gupta, S.C. and Kapoor, V. K. (1983). Fundamentals of Mathematical statistics.
- 4. Hogg, R. V. and Craig R. G. (1989). Introduction to Mathematical Statistics, MacMillan Publishing Co., NewYork.
- 5. Lefebvre Mario (2006). Applied probability and Statistics, Publisher Springer

- 6. Mayer, P. (1972). Introductory Probability and Statistical Applications, Addison Wesley Publishing Co., London.
- 7. Mood, A. M. and Graybill, F. A. and Boes D.C. (1974). Introduction to the Theory of Statistics, 3<sup>rd</sup> edition, McGraw Hill Book Company.
- 8. Mukhopadhyay P (2006) Mathematical Statistics Books & Allied (*P*) Ltd.
- 9. Rohatgi (2011). An Introduction to Probability and Statistics, Willey publications.
- 10. Ross S. (2002). A First Course in Probability, 6<sup>th</sup> edition, Pearson Education, Inc. & Dorling Kindersley Publishing,Inc.

Links: https://mahades. mamarashtra.gov.in

www.mospi.gov.in

http://www.isical.ac.in

https://iipsindia.org

### Progressive Education Society's Modern College of Arts, Science and Commerce (Autonomous), Shivajinagar, Pune – 5

Second Year of B. Sc. Statistics (NEP) (Minor)

Course Code: 23ScStaU4301 (Practical) Course Name: Probability Distributions

Teaching Scheme: 4 hours / Week Credit: 02

Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks

**Course Outcome**: At the end of this course students are expected to be able :

1. To compute probabilities of discrete probability distributions.

- 2. To identify appropriate discrete probability distribution to real life situations.
- 3. To apply discrete probability distribution to real life situations.
- 4. Model sample from various discrete probability distributions.
- 5. Study the probability distributions by studying the nature of probability curves.
- 6. Fit appropriate discrete probability distribution to real life situations.

	Title of Experiment/ Practical
1	Applications of Uniform Distribution.
2	Applications of Binomial Distribution.
3	Applications of Poisson Distribution.
4	Applications of Geometric and Negative Binomial Distribution.
5 Fitting of Discrete uniform Distribution.	
6	Fitting of Binomial Distribution and computation expected frequencies.
7	Fitting of Poisson Distribution and computation expected frequencies
8	Fitting of Geometric Distribution computation expected frequencies.
9	Fitting of negative binomial Distribution and computation expected frequencies.
10	Model Sampling from Uniform and Binomial Distribution
11	Model Sampling from Poisson Distribution
12	Sketching of probability distributions.
13	Project (equivalent to three practical's)

# Progressive Education Society's Modern College of Arts, Science and Commerce, Shivajinagar, Pune – 05 Second Year B.Sc. Course under NEP 2020 (OE)

Course Code: 23ScStaU4401 Course Name: Applied Statistics - II

Teaching Scheme: 2 hours/week Credits: 02

**Examination Scheme: CIA: 20 Marks End Sem: 30 Marks** 

**Course Outcomes:** At the end of this course students will be able to:

- 1. Understand Time Series Components. Interpret of Time Series Patterns:
- 2. Study methods of Time Series, Apply Time Series Analysis:
- 3. Understand Queuing Model Concepts, Probability Distribution and Queuing Parameters.
- 4. Interpret and Analyse Queuing Metrics.
- 5. Understand Vital Events and Statistics:
- 6. Analyse Trends in Vital Rates, Interpret Vital Rates.

Unit No.	Title	
Unit1	Exponential and Normal distribution	06
	1.1 Introduction of exponential distribution, only Statement of mean and	
	variance, lack of memory property, Applications of exponential distribution.	
	1.2 Introduction of normal distribution, only Statement of mean and variance,	
	Applications of normal distribution.	
Unit 2	Time Series	12
	2.1 Meaning and Utility.	
	2.2 Components of Time Series.	
	2.3 Additive and Multiplicative models.	
	2.4 Methods of estimating trend: moving Average method	
	2.5 Methods of estimating seasonality: simple average method	
	2.6 Cyclical variation- definition, distinction from seasonal variation,	
	Irregular variation- definition, illustrations.	
Unit 3	Vital Statistics	12
	3.1 Vital events, vital statistics, methods of obtaining vital statistics, rates of	
	vital events, sex ratios, dependency ratio.	
	3.2 Death/Mortality rates: Crude death rate, specific (age, sex etc.) death rate,	

- standardized death rate (direct and indirect), infant mortality rate.
- 3.3 Fertility/Birth rate: Crude birth rate, general fertility rate, specific (age, sex etc.) fertility rates, total fertility rate.
- 3.4 Growth/Reproduction rates Gross reproduction rate, net reproduction rate.
- 3.5 Interpretations of different rates, uses and applications.

- 1. Continuous probability distributions, (Second year B.Sc.) by Dr. P.G. Dixit, P.S.Kapre, Nirali Publication.
- 2. Fundamentals of Mathematical Statistics, by Gupta and Kapoor, Sultan Chand and Sons, New Delhi.
- 3. Discrete probability distributions, time series (Second year B.Sc.) by Dr. P.G. Dixit, P.S.Kapre, Nirali Publication.
- 4. Fundamentals of Applied Statistics, by Gupta and Kapoor, Sultan Chand and Sons, New Delhi.

## Progressive Education Society's Modern College of Arts, Science and Commerce, Shivajinagar, Pune – 5 Second Year of B.Sc. 2023

Second Year of B.Sc. 2023 (Course under NEP 2020)

Course Code: 23ScStaU4601 (Practical) Course Name: R programming

Teaching Scheme: 4 Hours/Week Credit: 2

Examination Scheme: CIA: 20 Marks End-Sem: 30 Marks

**Course Outcomes:** On completion of the course, student will be able to:

1. Understand the basics of R software

- 2. Identify time series component present in the data using R programming.
- 3. Use of R software for fitting of binomial distribution.
- 4. Use of R software for computation of descriptive statistics.
- 5. Write R code for mathematical computing.
- 6. Write R code for different sting operations.

### **Course Contents:**

Sr. No	Title	No. of
		Practical
1	Sketching pdf, cdf of a continuous random variables and	1
	computation quartiles, deciles, percentiles and model value)	
2	Application of exponential and normal distribution.	1
3.	Model sampling and fitting of exponential distribution.	1
4.	Model sampling and fitting of normal distribution.	1
5.	Application of central limit theorem	1
6.	Computations of probability from continuous uniform,	1
	normal, exponential, gamma, chi-square, t and F distribution.	
7.	Test based on normal, chi-square, T and F distribution	1
8.	Computation of birth rates or fertility rates	1
9.	Computation of death rates or mortality rates	1
10.	Computation of population growth rates	1
11.	Unweighted and weighted index number (laspryre's,	1
	Paasche's and Fisher index number) (Price, Quantity and	
	Value index number)	
12.	Cost of living index number, base shifting, splicing, deflating	1
13	Test of consistency and commonly used index numbers. (	
	Time reversal test, Factor reversal test and circular test)	
14-15	Experiential learning	2

- 1. Statistics using R (2<sup>nd</sup> edition) (Narosa Publication) by Sudha Purohit, Sharad Gore and Shailaja R. Deshmukh.
- 2. Statistical computing using R Software (Nirali publication ) by prof. Pawgi Viswas.